ON THE POSITIONAL UNCERTAINTY OF MULTI-ROBOT COOPERATIVE LOCALIZATION

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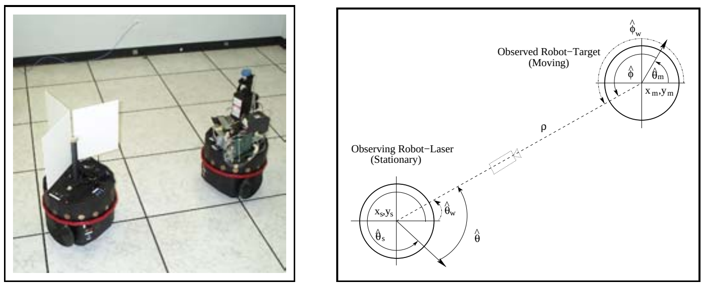
Abstract This paper deals with terrain mapping and position estimation using multiple robots. Here we will discuss work where a larger group of robots can mutually estimate one another's position (in 2D or 3D) and uncertainty using a sample-based (particle filter) model of uncertainty. Our prior work has dealt with a pair of robots that estimate one another's position using visual tracking and coordinated motion and we extend these results and consider a richer set of sensing and motion options. In particular, we focus on issues related to confidence estimation for groups of more than two robots.

Keywords: Cooperative Localization, Multi-Robot Navigation, Position Estimation, Localization, Mapping.

1. Introduction

In this paper we discuss the benefits of *cooperative localization* for a team of mobile robots. The term *cooperative localization* describes the technique whereby the members of a team of robots estimate one another's positions. This is achieved by employing a special sensor (*robot tracker*) that estimates a function of the pose of a moving robot relative to one or more stationary ones (see section 1.1). Furthermore, we consider the effects of different robot tracker sensors on the accuracy of localization for a moving robot *using only* the information from the rest of the robots (as opposed to observations of the environment). This approach results in an open loop estimate (with respect to the entire team) of the moving robot's pose without dependence on information from the environment. The experimental results allows us to examine the effectiveness of cooperative localization and estimate upper bounds on the error accumulation for different sensing modalities.



(a) (b)

Figure 1. (a) Two robots, one equipped with laser range finder (right) and the other with a target (left), employing cooperative localization. (b) Pose Estimation via Robot Tracker: Observation of the Moving Robot by the Stationary Robot. Note that the "camera" indicates the robot with the *Robot Tracker*; and $\hat{\theta}_w, \hat{\phi}_w$ are angles in world coordinates.

1.1 Cooperative Localization

Several different sensors have been employed for the estimation of the pose of one robot with respect to another robot. We restrict our attention to robot tracker sensors which return information in the frame of reference of the observing robot (i.e they estimate pose parameters relative to the robot making the observation). Consequently, for "two-dimensional robots" in a two dimensional environment, or for robots whose pose can be approximated as a combination of 2D position and an orientation, we can express the pose using three measurements; for ease of reference we represent them by the triplet $T = [\rho\ \phi\ \theta]$, where ρ is the distance between the two robots, ϕ is the angle at which the observing robot sees the observed robot relative to the heading of the observing robot, and θ is the heading of the observed robot as measured by the observing robot relative to the heading of the observing robot. (Figure 1b). If the stationary robot is equipped with the Robot Tracker, where $\mathbf{X}_m = [x_m, y_m, \theta_m]^T$ is the pose of the moving robot and $\mathbf{X}_s = [x_s, y_s, \theta_s]^T$ is the pose of the stationary robot then equation 1 returns the sensor output T:

$$\begin{bmatrix} \rho \\ \theta \\ \phi \end{bmatrix} = \begin{bmatrix} \sqrt{dx^2 + dy^2} \\ atan2(dy, dx) - \theta_s \\ atan2(-dy, -dx) - \theta_m \end{bmatrix} \begin{matrix} \text{Where :} \\ , dx = x_m - x_s \\ dy = y_m - y_s \end{matrix} \quad (1)$$

In order to estimate the probability distribution function (pdf) of the pose of the moving robot i at time t ($P(\mathbf{X}_i^t)$) we employ a particle filter (Monte Carlo simulation approach: see Jensfelt et al., 2000; Dellaert et al., 1999; Liu et al., 2001). The weights of the particles (W_i^t) at time t are updated using a Gaussian distribution (see equation 2 where $[\rho_i, \theta_i, \phi_i]^T$ has been calculated as in equation 1 but using the pose of particle "i" ($\mathbf{X}_{m_i}$) instead of the moving robot pose ($\mathbf{X}_m$)).

$$W_i^t = W_i^{t-1} \frac{1}{\sqrt{2\pi}\sigma_\rho} e^{\frac{-(\rho-\rho_i)^2}{2\sigma_\rho^2}} \frac{1}{\sqrt{2\pi}\sigma_\theta} e^{\frac{-(\theta-\theta_i)^2}{2\sigma_\theta^2}} \frac{1}{\sqrt{2\pi}\sigma_\phi} e^{\frac{-(\phi-\phi_i)^2}{2\sigma_\phi^2}} \qquad (2)$$

The rest of the paper is structured as follows. The next Section 2 presents some background work. Section 3 contains an analysis and experimental study of the primary different classes of sensory information that can be naturally used in cooperative localization. Finally, Section 4 presents our conclusions and a brief discussion of future work.

2. Previous Work

Prior work on multiple robots has considered collaborative strategies when the lack of landmarks made it impossible otherwise (Dudek et al., 1996). A number of authors have considered pragmatic multi-robot map-making. Several existing approaches operate in the sonar domain, where it is relatively straightforward to transform observations from a given position to the frame of reference of the other observers thereby exploiting structural relationships in the data (Leonard and Durrant-Whyte, 1991; Fox et al., 1998; Burgard et al., 2000). One approach to the fusion of such data is through the use of Kalman Filtering and its extensions (Roumeliotis and Bekey, 2000b; Roumeliotis and Bekey, 2000a).

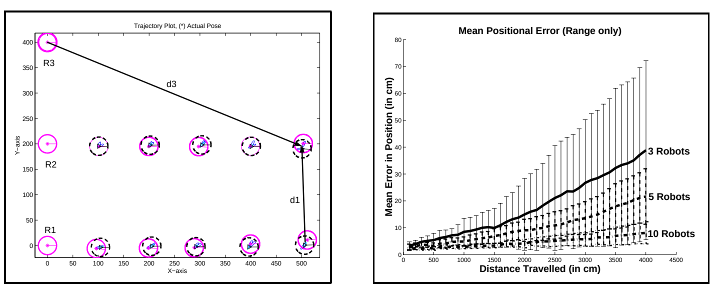
In other work, Rekleitis, Dudek and Milios have demonstrated the utility of introducing a second robot to aid in the tracking of the exploratory robot's position (Rekleitis et al., 2000). In that work, the robots exchange roles from time to time during exploration thus serving to minimize the accumulation of odometry error. The authors refer to this procedure as *cooperative localization.*

Recently, several authors have considered using a team of mobile robots in order to localize using each other. A variety of alternative sensors has been considered. For example, Kato et al., 1999 use robots equipped with omnidirectional vision cameras in order to identify and localize each other. In contrast, Davison and Kita, 2000 use a pair of robots, one equipped with an active stereo vision and one with active lighting to localize. The various methods employed for localization use different sensors with different levels of accuracy; some are able to estimate accurately the distance between the robots, others the orientation (azimuth) of the observed robot relative to the observing robot and some are able to estimate even the orientation of the observed robot.

3. Sensing Modalities

As noted above, several simple sensing configurations for a robot tracker are available. For example, simple schemes using a camera allow one robot to observe the other and provide different kinds of positional constraint such as the distance between two robots and the relative orientations. Moreover the group size affects the accuracy of the localization.

In the next part we present the effect the group size has on the accuracy of the localization for different sensors. The experimental arrangement of the robots is simulated and is consistent across all the sensing configurations. The robots start in a single line and they move abreast one at a time, first in ascending order and then in descending order for a set number of exchanges. The selected robot moves for 5 steps and after each step cooperative localization is employed and the pose of the moving robot is estimated. Each step is a forward translation by 100cm. Figure 2a presents a group of three robots, after the first robot has finished the five steps and the second robot performs the fifth step.



(a) (b)

Figure 2. (a) Estimation of the pose of robot R2 using only the distance from robot R1 (d1) and from robot R3 (d3). (b) Average error in position estimation using the distance between the robots only (3,4 and 10 robots; bars indicate std. deviation).

3.1 Range Only

One simple method is to return the relative distance between the robots. Such a method has been employed by Grabowski and Khosla, 2001 in the millibots project where an ultra-sound wave was used in order to recover the relative distance. In order to recover the position of one moving robot in the frame of reference of another, at least two stationary robots (that are not collinear with the moving one) are needed thus the minimum size of the group using this scheme is three robots.

Estimating the distance between two robots is very robust and relatively easy. In experimental simulations, the distance between every pair of robots was estimated and Gaussian, zero mean, noise was added with $\sigma_\rho = 2cm$ regardless the distance between the two robots. Figure 2b presents the mean error per unit distance traveled for all robots, averaged over 20 trials. As can be seen in Figure 2b with five robots, the positional accuracy is acceptable with an error of 20cm after 40m traveled; for ten robots the accuracy of the localization is very good.

3.2 Azimuth (Angle) Only

Several robotic systems employ an omnidirectional vision sensor that reports the angle at which another robot is seen. This is also consistent with information available from several types of observing systems based on pan-tilt units. In such cases orientation at which the moving robot is seen can be recovered with high accuracy. We performed a series of trials using only the angle at which one robot is observed, using groups of robots of different sizes. As can be seen in Figure 3 the accuracy of the localization does not improve as the group size increases. This is not surprising because small errors in the estimated orientation of the stationary robots scale non-linearly with the distance. Thus after a few exchanges the error in the pose estimation is dominated by the error in the orientation of the stationary robots.

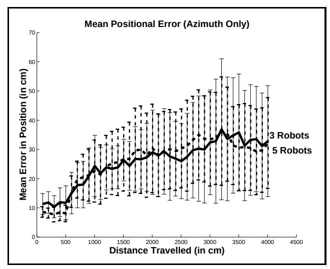
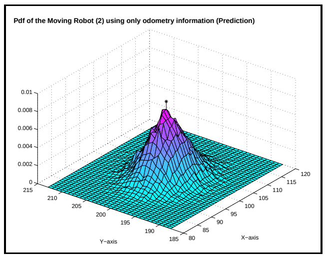
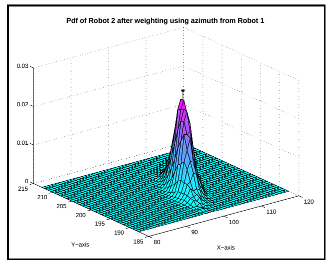


Figure 3: Average error in position estimation using the orientation of the moving robot is seen by the stationary ones.,

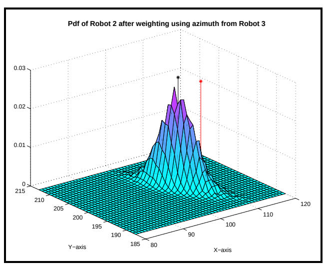
To illustrate the implementation of the particle filter, we present the probability distribution function (pdf) of the pose of the moving robot after one step (see Figure 4). The robot group size is three and it is the middle robot R2 that moves. The predicted pdf after a forward step can be seen in the first subfigure (4a) using odometry information only; the next two subfigures (4b,4c) present the pdf updated using the orientation at which the moving robot is seen by a stationary one (first by robot R1 then by robot R3); finally, the subfigure 4d presents the final pdf which combines the information from odometry and the observations from the two stationary robots. Clearly the uncertainty of the robot's position is reduced with additional observations.



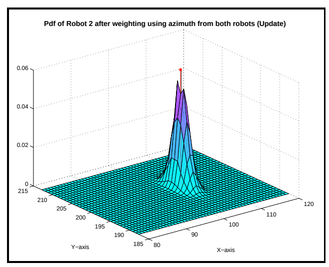
(a)



(b)

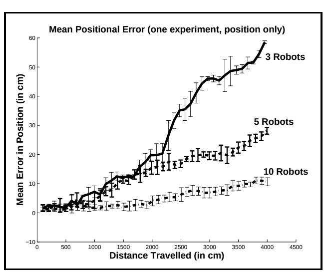


(c)

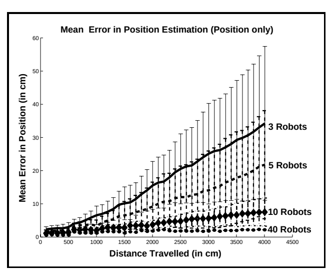


(d)

Figure 4. The pdf of the moving robot (R2) at different phases of its estimation: (a) prediction using odometry only; (b) using the orientation from stationary robot R1; (c) using the orientation from stationary robot R3; (d) final pdf.



(a)



(b)

Figure 5. Average error in position estimation using both the distance between the robots and the orientation the moving robot is seen by the stationary ones. (a) Average error in positioning of the team of robots one trial (3,5 and 10 robots). (b) Average error in position estimation over twenty trials (3,5, 10 and 40 robots).

3.3 Position Only

Another common approach is to use the position of one robot computed in the frame of reference of another (relative position). This scheme has been employed with two robots (see Burgard et al., 2000) in order to reduce the uncertainty. The range and azimuth information ($[\rho, \theta]$) is combined in order to improve the pose estimation. As can be seen in Figure 5a even with three robots the error in pose estimation is relatively small (average error 30cm for 40m distance traveled per robot, or 0.75%). In our experiments the distance between the two robots was estimated and, as above, zero-mean Gaussian noise was added both to distance and to orientation with $\sigma_\rho = 2cm$ and $\sigma_\theta = 0.5°$ respectively. The experiment was repeated twenty times and the average error in position is shown in Figure 5b for groups of robots of size 3,5,10 and 40.

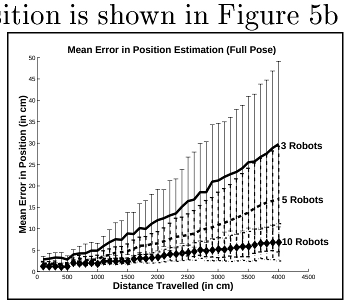


Figure 6. Average error in position estimation using full pose $[\rho, \theta, \phi]$.

# of Robots	3	5	10
Range (ρ)	38.80	21.63	8.13
Azimuth (θ)	32.83	32.20	
Position (ρ, θ)	34.25	21.79	7.50
Full Pose (ρ, θ, ϕ)	28.73	16.72	6.08

Table 1. The mean error in position estimation after 40m travel over 20 trials.

3.4 Full Pose

Some robot tracker sensors provide accurate information for all three parameters $[\rho, \theta, \phi]$ and they can be used to accurate estimate the full pose of the moving robots (see Kurazume and Hirose, 1998; Rekleitis et al., 2001). In the experimental setup the robot tracker sensor was characterized by Gaussian, zero mean, noise with $\sigma = [2cm, 0.5°, 1°]$. By using the full equation 2 we weighted the pdf of the pose of the moving robot and performed a series of experiments for 3, 5 and 10 robots; with very low positional error (see Figure 6).

4. Conclusions

In this work we examined the effect of the size of the team of robots and the sensing paradigm on cooperative localization (see Table 1 for a synopsis). Also, preliminary results from experiments with varying odometry error have shown that cooperative localization is robust even with 10-20% odometry errors.

In future work we hope to further extend the uncertainty study for different group configurations and motion strategies. An interesting extension would be for the robots to autonomously develop a collaborative strategy to improve the accuracy of localization (see Potter et al., 2001). Given a large group of robots, an estimate of the effects of team size on error accumulation would allow the group of be effectively partitioned to accomplish sub-tasks while retaining a desired level of accuracy in positioning.

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